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Hyperon scalar form factors

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Abstract. We estimate, by means of a model, the long-range part of the hyperon scalar form factors and their contribution to the pion–hyperon sigma term.

1 Introduction

High-energy proton–nucleus or nucleus–nucleus collisions may release large amounts of energy into very small regions and disturb strongly the QCD vacuum. After hadronization, these reactions produce final states which, typically, contain many pions and a wide variety of other particles, including strange ones. Pion–hyperon (πY) interactions, therefore, are among the many elements that contribute to the detailed description of high-energy collisions. The fact that these particles are produced in the same reaction allows one to assume that they interact as comovers in an expanding system, with relative energies which are not very high, considering that in these cases the freeze-out temperature is of the order of the pion mass.

An interesting feature of high-energy proton–nucleus collisions is that both hyperons and antihyperons detected in inclusive processes may be polarized [1]. In 1993, Hama and Kodama [2] assumed that antihyperons were produced unpolarized and would become polarized afterwards, by interacting with the surrounding particles. They employed an optical potential and found out that it had to depend on the particular hyperon considered. So, in high-energy pA collisions, antihyperon polarization can be understood by means of hyperon dependent final state interactions.

This idea was further developed recently [4,5] using the hydrodynamical model, which describes well many of the main feactures of the high-energy multiparticle production, supplemented by a microscopic (low-energy) chiral pion–hyperon interaction [6].

In low-energy πY scattering, the polarization is given by

$$\boldsymbol{P} = -2\frac{\mathrm{Im}(f^* g)}{|f|^2 + |g|^2}\ \hat{n}, \qquad (1)$$

where f and g are the usual [7] spin no-flip and spin flip amplitudes, given by

$$f \sim f_S + (2f_{P3} + f_{P1})\cos\theta,$$

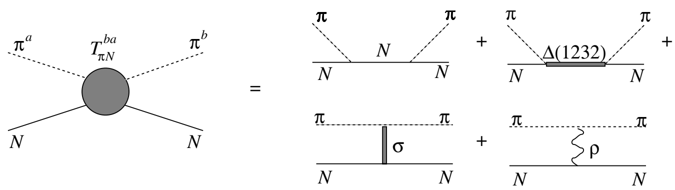

Fig. 1. πN interaction

$$g \sim (f_{P3} - f_{P1})\sin\ \theta, \qquad (2)$$

where $\hat{n}$ is the normal to the reaction plane and θ is the scattering angle. Polarization is thus sensitive to the interference between S and P waves. Details on low-energy polarization may be found in [4,6], and on high-energy polarization in [4,5]. As there are no phase shift analyses for πY scattering, one needs a model in order to describe this process. In [6] we calculated the polarization using a chiral model adapted from low-energy πN interactions [8], shown in Fig. 1, where amplitudes are saturated by spin 1/2 and 3/2 intermediate states, supplemented by the exchange of a scalar system in the t-channel. This last contribution has a strong influence over the amplitude f_S in (2) and hence plays a determinant role in the polarization. It is related, by means of the Ward–Takahashi identity, to the scalar form factor of the baryon, denoted by $\sigma(t)$. At large distances, this function is dominated by triangle diagrams involving the exchange of two pions.

In the πN model of [8], the dependence of the scalar exchange on t, at low energies, had the form $a_N + b_N t$, and the values of the parameters (a_N, b_N) could be extracted from empirical subthreshold coefficients. By analogy, in [6], the πY scalar exchange was assumed to be given by $a_Y + b_Y t$. In [3] we found that one needed to use values such that $(a_\Xi, b_\Xi) < (a_\Sigma, b_\Sigma) < (a_\Lambda, b_\Lambda) < (a_N, b_N)$, as g_A and g_{A^*}, considering $SU(3)$, decrease with the hyperon mass (see Table 1), but this ansatz remained unproved.

These findings motivate the present work, in which we try to justify this hierarchy by evaluating the pion cloud contribution to the scalar form factor of spin 1/2 hyperons. Our calculation follows a procedure used previously in the nucleon case [9].

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Table 1. Axial coupling constants for the processes $\pi B \to B'(g_A)$ and $\pi B \to R(g_A^*)$

	N	Δ	Λ	Λ^* (1405)	Σ	Σ^* (1385)	Ξ	Ξ^* (1530)
N	1.25	2.82	–	–	–	–	–	–
Λ	–	–	–	–	0.98	1.74	–	–
Σ	–	–	0.98	1.63	0.52	~ 0	–	–
Ξ	–	–	–	–	–	–	0.28	0.84

The interactions of pions with other hadrons can be well described by means of effective theories, in which an approximate $SU(2) \times SU(2)$ symmetry is broken by the small pion mass (μ). In the framework of chiral perturbation theory, the leading term of the nucleon scalar form factor is proportional to μ^2 and determined directly by the coupling constant c_1 of the second order Lagrangian [10] (the second order Lagrangian shown in [10] contains four independent coupling constants, $c_1, \ldots, c_4$). Loop diagrams, which carry the dependence on t, begin contributing at order $\mathcal{O}(\mu^3)$. This means that, in a strict calculation, one is not able to predict the value of the parameter a, which is related with c_1. In order to overcome this difficulty, a model is needed.

One notes that, to $\mathcal{O}(\mu^3)$, triangle diagrams are completely determined, since they involve only known masses and coupling constants. To $\mathcal{O}(\mu^4)$, loop diagrams combine with the constants c_1, c_2 and c_3. The study of πN subthreshold coefficients indicates that c_1 is smaller than c_2 and c_3, which are saturated by the Δ. This allows the $\mathcal{O}(\mu^4)$ scalar form factor to be well represented by the leading tree contribution associated with c_1 and two triangle diagrams, involving N and Δ intermediate states, as in Fig. 1.

When one goes to configuration space, these two kinds of contributions split apart in $\tilde{\sigma}(r)$, the Fourier transform of $\sigma(t)$. The tree term yields a zero-range δ-function, whereas the triangle diagrams give rise to spatially distributed structures, fully determined by known parameters. As noted in [11], for the case of hadron polarizabilities, the Fourier transform acts as a filter which transmits only genuine pion loop effects. The σ-term, defined as $\sigma \equiv \sigma(t=0)$, is given by

$$\sigma = 4\pi \int_0^\infty \mathrm{d}r r^2 \tilde{\sigma}(r). \tag{3}$$

In non-linear lagrangians, the pion degrees of freedom are better described by a direction $\hat{\boldsymbol{\pi}}$ and by an angle θ, embodied into the operator $U = \exp(\mathrm{i}\boldsymbol{\tau} \cdot \hat{\boldsymbol{\pi}}\theta) = \cos\theta + \mathrm{i}\boldsymbol{\tau} \cdot \hat{\boldsymbol{\pi}} \sin\theta$. In this framework, the value of the dimensional pion field $\boldsymbol{\phi} = f_\pi \sin\theta \hat{\boldsymbol{\pi}}$ cannot be larger than f_π, since Goldstone bosons are collective states derived from the $q\bar{q}$ condensate. Accordingly, pion loop effects correspond to a transformation of the condensate that surrounds the nucleon and the corresponding energy densities cannot exceed $\mu^2 f_\pi^2$, that of the original condensate. We define a radius R by the relationship $\tilde{\sigma}(R) = f_\pi^2 \mu^2$ and replace (3) with

$$\sigma = \frac{4}{3}\pi R^3 f_\pi^2 \mu^2 + 4\pi \int_R^\infty \mathrm{d}r r^2 \tilde{\sigma}_\ell(r), \tag{4}$$

where $\tilde{\sigma}_\ell(r)$ is the loop density. In the case of the nucleon [9], this expression yields $\sigma = 46\,\mathrm{MeV}$, a value quite close to that prescribed in [12]. In Appendix B we discuss the relationship of this approach with the usual one [10] and show that the cutting radius can be interpreted as a kind of renormalization scale. These results support the present extension of this procedure to the case of strange baryons.

2 Formalism

The scalar form factor for a spin 1/2 baryon B is defined as $\langle B(p')| - \mathcal{L}_{sb}|B(p)\rangle \equiv \sigma(t)\bar{u}(\boldsymbol{p}')u(\boldsymbol{p})$, where $\mathcal{L}_{sb}$ is the chiral symmetry breaking lagrangian and $t = (p-p')^2$. In this work we assume the scalar form factor of strange baryons to be given by the processes shown in Fig. 1. The relevant interaction Lagrangians are given by

$$\mathcal{L}_{\pi BB'} = \frac{g_A}{2f_\pi}\left[\bar{B}'\gamma_\mu\gamma_5 T_a B\right]\partial^\mu\phi_a + \text{h.c.}, \tag{5}$$

and

$$\mathcal{L}_{\pi BR} = \frac{g_A^*}{2f_\pi}\left[\bar{R}_\mu T_a B\right]\partial^\mu\phi_a + \text{h.c.}, \tag{6}$$

where B, R and ϕ denote respectively spin 1/2, spin 3/2 and pion fields[1], T is a matrix that couples baryons into an isospin 1 state and f_π is the pion decay constant. The coupling constants g_A and g_A^* for the processes $\pi B \to B'$ and $\pi B \to R$, are given in Table 1. The former were obtained from the usual $SU(3)$ relations [13] using the πNN and $\pi\Lambda\Sigma$ [14] vertices as input. One notes that they would not change much if the more recent results of [15]

[1] In analogy with the $\pi N\Delta$ coupling, the πBR lagrangian can be written in a more general form [8] as

$$\mathcal{L}_{\pi BR} = \frac{g_A^*}{2f_\pi}\left\{\bar{R}_\mu\left[g^{\mu\nu} - \left(Z - \frac{1}{2}\right)\gamma^\mu\gamma^\nu\right]T_a B\right\}\partial^\mu\phi_a + \text{h.c.}$$

However, the influence of the parameter Z over πN subthreshold coefficients amounts to just a few percent and we restrict ourselves to the simpler form given in (6) (using $Z = -0.5$, the accepted value for the πN interaction) in this exploratory work.

were used. The values of g_A^* were taken from Breit–Wigner fits to resonance decay widths [6]. The spin 3/2 propagator for a particle of mass M is written as

$$G^{\mu\nu}(p) = -\frac{(\not p + M)}{p^2 - M^2} \times \left(g^{\mu\nu} - \frac{\gamma^\mu\gamma^\nu}{3} - \frac{\gamma^\mu p^\nu}{3M} + \frac{p^\mu\gamma^\nu}{3M} - \frac{2p^\mu p^\nu}{3M^2} \right). \quad (7)$$

The initial and final baryon momenta are denoted by p and p', whereas k and k' are the momenta of the exchanged pions. We also use the variables $P = (p'+p)/2$, $q = k'-k$, $Q = (k+k')/2$, $t = q^2$ and $s = [Q^2 + 2P\cdot Q - t/4 + m^2]$.

The contribution of an intermediate particle of spin s and mass M to the scalar form factor is given by

$$\sigma_s(t;M)\bar u u = \mathrm{i}\mu^2 \left(\frac{g_A}{2f_\pi}\right)^2 \left(T_a^\dagger T_a\right) \int [\ldots][\bar u \Lambda_s u], \quad (8)$$

with

$$\int [\ldots] = \int \frac{\mathrm{d}^4 Q}{(2\pi)^4} \frac{1}{[(Q-q/2)^2 - \mu^2][(Q+q/2)^2 - \mu^2]}, \quad (9)$$

$$[\bar u \Lambda_{1/2} u] = \bar u \left\{ -(m+M) + \frac{(m-M)(m+M)^2}{s-M^2} + \left[1 + \frac{(m+M)^2}{s-M^2}\right] \not Q \right\} u, \quad (10)$$

$$\begin{aligned}[\bar u \Lambda_{3/2} u] = -\bar u \Bigg\{ &\left[\frac{1}{s-M^2} \left((m+M)(\mu^2 - t/2) - \frac{(2M+m)}{6M^2}\mu^4 \right) \right. \\ &+ \left(\frac{m^2-M^2}{s-M^2} - 1 \right) \frac{(m+M)}{6M^2} \\ &\left. \times \left((m+M)(2M-m) + 2\mu^2 \right) - \frac{m(s-m^2)}{6M^2} \right] \\ &+ \left[\frac{1}{s-M^2} \left((\mu^2 - t/2) + \frac{2m}{3}(m+M) - \frac{(m+M)\mu^2}{3M} - \frac{\mu^4}{6M^2} \right) + \left(\frac{m^2-M^2}{s-M^2} - 1 \right) \right. \\ &\left. \times \frac{1}{6M^2}\left(M^2 + 2mM - m^2 + 2\mu^2\right) - \frac{s-m^2}{6M^2} \right] \not Q \Bigg\} u. \end{aligned} \quad (11)$$

In writing these expressions we have replaced k^2 and k'^2 in the numerator with μ^2. This approximation amounts to neglecting short-range interactions, since terms proportional to $(k^2 - \mu^2)$ and $(k'^2 - \mu^2)$ in the numerator may be used to cancel pion propagators in (8).

Using the loop integrals Π defined in Appendix A, we obtain

$$\begin{aligned}\sigma_{1/2}(t) = \frac{T_a^\dagger T_a}{(4\pi)^2} \left(\frac{g_A \mu}{2f_\pi}\right)^2 \frac{(m+M)}{8m^3} \Big\{ &[4(m-M)m^2 - t(m+M)]\, \Pi_{cc}^{(000)} \left[-4(m-M)m^2 + t(m+M) \right. \\ &\left. + 4m^2(\mu^2 - t/2)/(m-M) \right] \frac{m^2-M^2}{2m\mu} \Pi_{\bar s c}^{(000)} \Big\}, \end{aligned} \quad (12)$$

$$\begin{aligned}\sigma_{3/2}(t) = \frac{T_a^\dagger T_a}{(4\pi)^2} \left(\frac{g_A^* \mu}{2f_\pi}\right)^2 \frac{(m+M)}{24mM^2} \times \Bigg\{ &\Bigg[2(m^2-M^2)(m+M) + \frac{(\mu^2 - t/2)}{m^2(m+M)} \\ &\times \left(M^4 - 2mM^3 + 6m^2M^2 - 2m^3M - \frac{13m^4}{3} \right) \\ &+ \frac{\mu^2}{m^2}\left(-M^3 + 3mM^2 - 5m^2M - 5m^3 - \frac{4m^4}{3(m+M)} \right) \Bigg] \\ &\times \Pi_{cc}^{(000)} \\ &+ \Bigg[-2(m^2-M^2)(m+M) + 8(\mu^2 - t/2)\frac{mM^2}{m^2-M^2} + \frac{(\mu^2 - t/2)}{m^2(m+M)} \\ &\times \left(-M^4 + 2mM^3 - 8m^2M^2 - 2m^3M + m^4 \right) \\ &+ \frac{\mu^2}{m^2}\left(M^3 - 3mM^2 + 5m^2M + 5m^3 \right) \Bigg] \\ &\times \frac{(m^2-M^2)}{2m\mu} \Pi_{\bar s c}^{(000)} \Bigg\}, \end{aligned} \quad (13)$$

with

$$\begin{aligned}\Pi_{cc}^{(000)} &= -\int_0^1 \mathrm{d}a \ln\left(1 + a(a-1)t/\mu^2\right) \\ &= 1 - \sqrt{1 - 4\mu^2/t} \ln\left[\frac{\sqrt{1-4\mu^2/t} + 1}{\sqrt{1-4\mu^2/t} - 1} \right], \end{aligned} \quad (14)$$

$$\begin{aligned}\Pi_{\bar s c}^{(000)} &= (-2m/\mu) \int_0^1 \mathrm{d}a a \\ &\times \int_0^1 \mathrm{d}b \mu^2 / \left(a(1-a)(1-b)t \right. \\ &\left. + [\mu^2 - ab(\mu^2 + m^2 - M^2) + a^2b^2m^2] \right). \end{aligned} \quad (15)$$

For $M = m$, one has $\Pi_{cc}^{(000)} \sim \Pi_{\bar s c}^{(000)} \sim \mathcal{O}(\mu^0)$ and hence the loop contribution to $\sigma(t)$ is $\mathcal{O}(\mu^3)$, as expected. This result does not change when $M \neq m$ because, from (A.8), one learns that $[\Pi_{cc}^{(000)} - (m^2 - M^2)\Pi_{\bar s c}^{(000)}/2m\mu] \sim \mathcal{O}(\mu^3)$.

The scalar form factor in configuration space is obtained by going to the Breit frame and writing

$$\tilde\sigma(r) = \int \frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} \mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} \sigma(t). \quad (16)$$

The corresponding expressions are derived from (12) and (13) through the replacements $t \to \boldsymbol{\nabla}^2$ and $\Pi \to \mu^3 S$, with S given by (A.10)–(A.11).

3 Results and conclusions

We begin by discussing our results for the ratios $\tilde{\sigma}(r)/f_\pi^2\mu^2$ for the spin 1/2 baryons. In Fig. 2 we display the individual contributions of the various intermediate states as functions of the distance and one notes that the role of resonances is rather important. Comparing this feature with the fact that, in the framework of chiral symmetry, one has $\sigma_{1/2} \to \mathcal{O}(\mu^3)$ and $\sigma_{3/2} \to \mathcal{O}(\mu^4)$, one learns that the power counting hierarchy is subverted around $r \sim 1\,\mathrm{fm}$.

The full curves for the N, Λ, Σ and Ξ states are shown in Fig. 3. The values of the distance R for which $\tilde{\sigma}(R)/f_\pi^2\mu^2 = 1$ and of the σ-term, calculated by means of (4), are given in Table 2. One finds that heavier systems correspond to smaller values of these quantities, but one should bear in mind that the coupling constants of Table 1 also intervene.

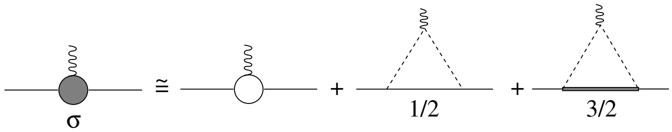

Fig. 2. The scalar form factor (grey blob) receives contributions from tree interactions (white blob) and triangle diagrams with spin 1/2 and 3/2 intermediate states

Table 2. Baryon radius (R), σ-term = $\sigma(0)$ and $\sigma(2\mu^2)$

	N	Λ	Σ	Ξ
R (fm)	0.58	0.51	0.45	0.35
σ (MeV)	46.0	33.5	29.2	12.0
$\sigma(2\mu^2)$ (MeV)	57.6	39.3	36.2	13.25

These results allow the scalar form factor in momentum space to be written, for each of the baryons considered, as

$$\sigma(t) = \sigma + \sum \left[\sigma_s(t; M) - \sigma_s(0)\right], \tag{17}$$

where the summation runs over possible intermediate states,

$$\begin{aligned}
&\sigma_N(t) + \sigma_N(0) = \sigma + \sigma_{1/2}(t; N) + \sigma_{3/2}(t; \Delta), \\
&\sigma_\Lambda(t) + \sigma_\Lambda(0) = \sigma + \sigma_{1/2}(t; \Lambda) + \sigma_{3/2}(t; \Sigma^*(1385)), \\
&\sigma_\Sigma(t) + \sigma_\Sigma(0) \\
&\quad = \sigma + \sigma_{1/2}(t; \Sigma) + \sigma_{1/2}(t; \Lambda) + \sigma_{1/2}(t; \Lambda^*(1405)), \\
&\sigma_\Xi(t) + \sigma_\Xi(0) = \sigma + \sigma_{1/2}(t; \Xi) + \sigma_{3/2}(t; \Xi^*(1530)),
\end{aligned} \tag{18}$$

with $\sigma_s(t; M)$ given in (12) and (13). The results are shown in Fig. 5.

In Table 2 we also quote $\sigma(2\mu^2)$, the value of this function at the Cheng–Dashen point. The contribution of $\sigma(t)$ to the isospin even πY scattering subamplitude A^+ [6] is given by $\sigma(t)/f_\pi^2$. It corresponds to the t-channel exchange of a scalar system. With the motivation discussed in the introduction, we make a linear numerical fit of $\sigma(t)$ at low energies,

$$\sigma(t)/f_\pi^2 = a + b\,t. \tag{19}$$

The coefficients given in Table 3, yield reasonable results for $t > -0.1$.

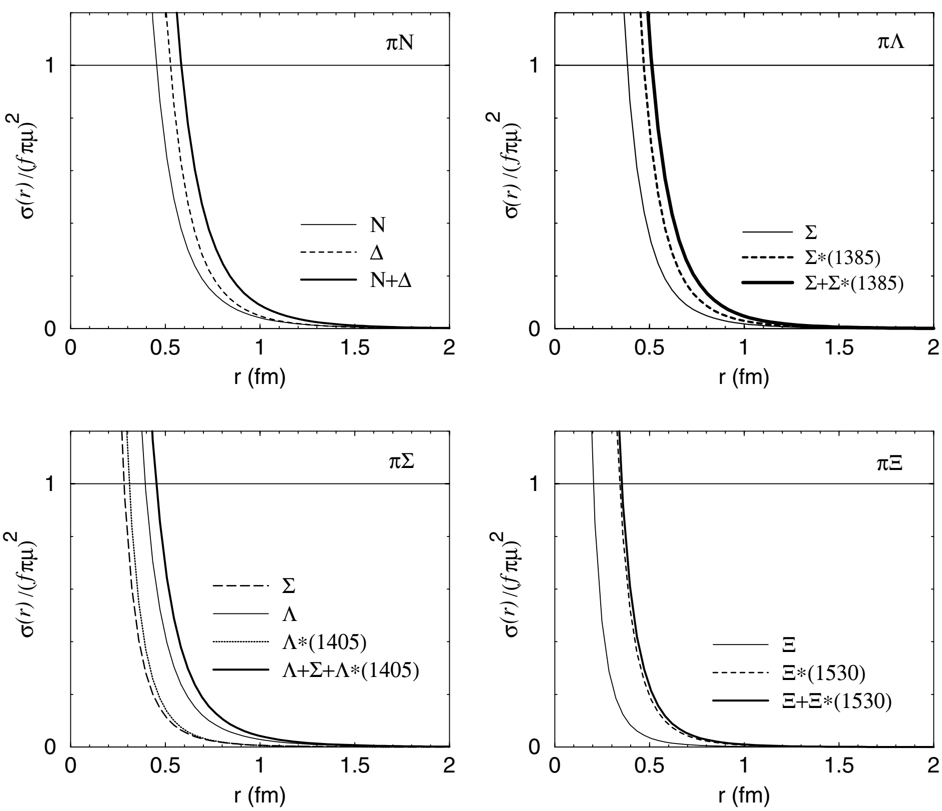

Fig. 3. Intermediate state contributions to the scalar form factor of the spin 1/2 baryons

Table 3. Coefficients of the series given in (19) and values used in [4], within brackets

	N	Λ	Σ	Ξ
$a(\mu^{-1})$	0.7423 [0.25]	0.5390 [0.22]	0.4698 [0.13]	0.1936 [0.07]
$b(\mu^{-3})$	0.0690 [0.40]	0.0361 [0.35]	0.0322 [0.20]	0.0074 [0.12]

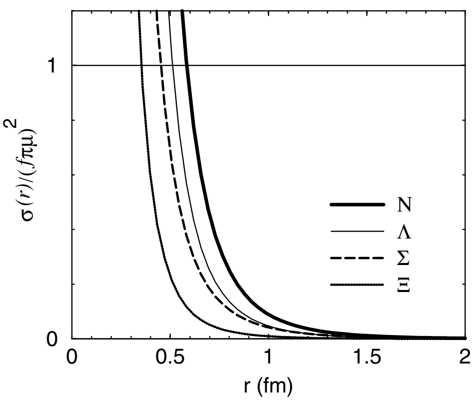

Fig. 4. Full results for the scalar form factor of the spin 1/2 baryons

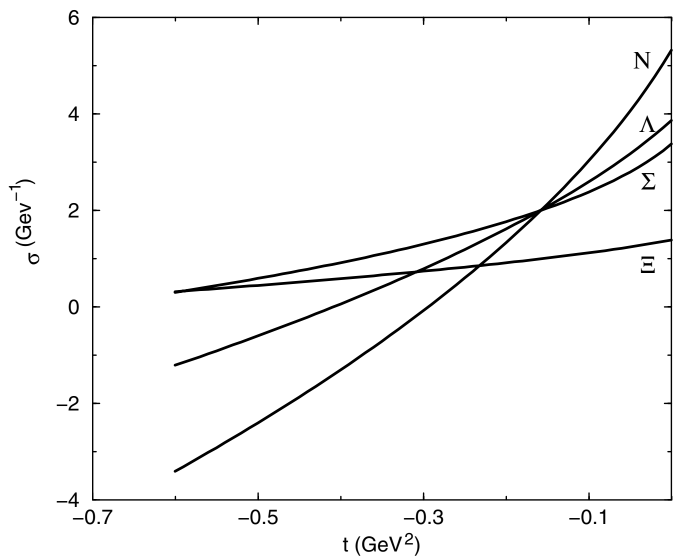

Fig. 5. $\sigma(t)$ for N, Λ, Σ and Ξ

These results support qualitatively the assumptions made in [6], namely that the hyperon parameters must be smaller than those of the nucleon. They also indicate that the values used in [4], given within brackets in Table 3, must be updated. In a previous work we have learned that the influence of the scalar form factor over high-energy polarization stays in the range 10–30% and hence a theoretical determination of this function in the case of hyperons may improve the reliability of inputs into hydrodynamical calculations. A full calculation of the hyperon polarization in high-energy proton–nucleus collisions based on the theoretical values discussed here is in progress and will be reported elsewhere.

Appendix A: Loop integrals

The basic loop integrals needed in this work are given by

$$I_{cc}^{\mu\dots} = \int [\dots]\,(Q^\mu/\mu\dots)\,, \tag{A.1}$$

$$I_{\bar{s}c}^{\mu\dots} = \int [\dots]\,(Q^\mu/\mu\dots\ )\,\frac{2m\mu}{[s-M^2]}. \tag{A.2}$$

All denominators are symmetric under $q \to -q$ and hence results cannot contain odd powers of this variable. The integrals are dimensionless and have the following tensor structure:

$$I_{cc} = \frac{\mathrm{i}}{(4\pi)^2}\left\{\Pi_{cc}^{(000)}\right\}, \tag{A.3}$$

$$I_{cc}^{\mu\nu} = \frac{\mathrm{i}}{(4\pi)^2}\left\{q^\mu q^\nu/\mu^2\,\Pi_{cc}^{(200)} + g^{\mu\nu}\,\bar{\Pi}_{cc}^{(000)}\right\}, \tag{A.4}$$

$$I_{\bar{s}c} = \frac{\mathrm{i}}{(4\pi)^2}\left\{\Pi_{\bar{s}c}^{(000)}\right\}, \tag{A.5}$$

$$I_{\bar{s}c}^{\mu} = \frac{\mathrm{i}}{(4\pi)^2}\left\{P^\mu/m\,\Pi_{\bar{s}c}^{(001)}\right\}. \tag{A.6}$$

These integrals are not independent. Multiplying (A.1) by q^μ, by $g^{\mu\nu}$ and neglecting short-range terms one has

$$\bar{\Pi}_{cc}^{(000)} = \frac{1}{3}\left(1 - t/4\mu^2\right)\Pi_{cc}^{(000)} + \dots \tag{A.7}$$

Multiplying (A.2) by P^μ, one gets

$$\frac{P^2}{m^2}\Pi_{\bar{s}c}^{(001)} \tag{A.8}$$
$$= \Pi_{cc}^{(000)} - \left[\left(\mu^2 - t/2\right) + (m^2 - M^2)\right]\frac{\Pi_{\bar{s}c}^{(000)}}{2m\mu} + \dots$$

The dimensionless configuration space functions S are defined as

$$S = \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\,\Pi, \tag{A.9}$$

with $x = \mu r$ and $\boldsymbol{k} = \boldsymbol{q}/\mu$. One has

$$S_{cc}^{(000)} = \frac{4}{4\pi}\,\frac{K_1(2x)}{x^2}, \tag{A.10}$$

$$S_{\bar{s}c}^{(000)} = -\frac{1}{4\pi x}\int_0^1 \mathrm{d}a a\int_0^1 \mathrm{d}b\,\frac{2m/\mu}{a(1-a)(1-b)}\,\mathrm{e}^{-x\theta_{\bar{s}c}}, \tag{A.11}$$

where

$$\theta^2_{\bar{s}c} = \frac{1 - ab[1 + (m^2 - M^2)/\mu^2] + a^2 b^2 (m/\mu)^2}{a(1-a)(1-b)}. \quad \text{(A.12)}$$

For the nucleon, the following approximation holds [10]:

$$S^{(000)}_{\bar{s}c} = -\frac{\mathrm{e}^{-2x}}{2\,x^2} + \frac{\mu}{\pi m x}\left[K_0(2x) + \frac{K_1(2x)}{x}\right] - \frac{\mu^2}{8m^2}\frac{\mathrm{e}^{-x}}{x}. \quad \text{(A.13)}$$

Appendix B: Cutting radius as renormalization scale

In the case of the nucleon, (12) becomes

$$\sigma = \frac{4}{3}\pi R^3 f_\pi^2 \mu^2 + \frac{3}{16\pi}\frac{g_A^2}{f_\pi^2}\mu^3 \times \int_\rho^\infty \mathrm{d}x\; x^2 \left\{\left(1 - \frac{\boldsymbol{\nabla}^2}{2}\right) S^{(000)}_{\bar{s}c} - \frac{\mu}{2m}\boldsymbol{\nabla}^2 S^{(000)}_{cc}\right\}, \quad \text{(B.1)}$$

where $\rho = \mu R$ and the functions S are given by (A.10) and (A.13). All integrals can be performed analytically and one has

$$\sigma = \frac{4}{3}\pi R^3 f_\pi^2 \mu^2 + \frac{3\,g_A^2}{16\pi f_\pi^2}\mu^3 \left\{\left[\frac{1}{4} + \frac{1}{2\rho}\right]\mathrm{e}^{-2\rho} + \frac{\mu}{2\pi m}\left[2K_0(2\rho) - \rho K_1(2\rho) - 6K_2(2\rho)\right] + \frac{\mu^2}{16m^2}\left(\frac{1}{2} + \rho\right)\mathrm{e}^{-2\rho}\right\}. \quad \text{(B.2)}$$

The chiral expansion of this result yields

$$\sigma = \left[\frac{4}{3}\pi R^3 f_\pi^2 + \frac{3}{32\pi}\frac{g_A^2}{f_\pi^2}\left(\frac{1}{R} + \frac{3}{\pi m R^3}\right)\right]\mu^2 - \frac{9}{64\pi}\frac{g_A^2}{f_\pi^2}\mu^3 - \frac{3\,g_A^2}{16\pi^2 f_\pi^2 m}\mu^4\left[\ln(\mu R) + \frac{m_\pi R}{2} + \gamma + \frac{5}{2}\right]. \quad \text{(B.3)}$$

Comparison with the standard result [10]

$$\sigma = -4c_1\mu^2 - \frac{9\,g_A^2\mu^3}{64\pi f_\pi^2} - \frac{3\,g_A^2\mu^4}{16\pi^2 f_\pi^2 m}\ln\left(\frac{\mu}{\Lambda}\right) - \frac{9\,g_A^2\mu^4}{64\pi^2 f_\pi^2\, m} \quad \text{(B.4)}$$

indicates that the structures of non-analytic terms are identical, provided one identifies the cutting radius R with the inverse of the renormalization scale Λ.

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